

Unit 6 Test Review

Date _____ Period _____

Identify the domain and range of each.

1) $y = \log_2(4x + 22) - 2$

D: $(-\frac{11}{2}, \infty)$

R: $(-\infty, \infty)$

$$\begin{aligned} 2^x &= \\ 4x + 22 &= 0 \\ -22 &= \\ \frac{-22}{4} &= \\ -\frac{11}{2} &= \end{aligned}$$

2) $y = \log_5(3x - 5) + 4$

D: $(\frac{5}{3}, \infty)$

R: $(-\infty, \infty)$

3x - 5 = 0

$\frac{5}{3}$

Solve each equation.

3) $243^{3p} = 81$

$(3^5)^{3p} = 3^4$

$\frac{15p}{15} = \frac{4}{15}$

$p = \frac{4}{15}$

4) $216^{-3n} = 36^n$

$6^{3(-3n)} = 6^{2n}$

$-9n = 2n$

$n = 0$

5) $\left(\frac{1}{243}\right)^{-p-2} = 27$

$(3^{-5})^{-p-2} = 3^3$

$-5(-p-2) = 3$

$\frac{5p+10}{-10} = \frac{3}{-10}$

$\frac{5p}{5} = \frac{-7}{5}$

6) $625^{3b} = \frac{1}{125}$

$(5^4)^{3b} = 5^{-3}$

$\frac{12b}{12} = -\frac{3}{12}$

$b = -\frac{1}{4}$

7) $\left(\frac{1}{9}\right)^{2-3v} = 81$

$3^{-2(2-3v)} = 3^4$

$-4 + 6v = 4$

$\frac{6v}{6} = \frac{8}{6}$

$v = \frac{4}{3}$

8) $9^{3b} \cdot \left(\frac{1}{27}\right)^{3b+1} = 9^b$

$(3^2)^{3b} \cdot 3^{-3(3b+1)} = 3^{2b}$

$6b + -9b - 3 = 2b$

$-3b - 3 = 2b$

$\frac{-3}{5} = \frac{5b}{5}$

$b = -\frac{3}{5}$

$$9) 8^{3n} \cdot \left(\frac{1}{2}\right)^{2n+2} = \frac{1}{64}$$

$$2^{3(3n)} \cdot 2^{-1(2n+2)} = 2^{-6}$$

$$9n + -2n - 2 = -6$$

$$\frac{7n}{7} = \frac{-4}{7} \quad n = \frac{-4}{7}$$

$$10) 8^{2n+2} \cdot \left(\frac{1}{64}\right)^{2-n} = 16$$

$$2^{3(2n+2)}, 2^{-6(2-n)} = 2^4$$

$$6n + 6 - 12 + 6n = 4$$

$$12n - 6 = 4$$

$$\frac{12n}{12} = \frac{10}{12} \quad n = \frac{5}{6}$$

Solve each equation. Round your answers to the nearest ten-thousandth.

$$11) \frac{5 \cdot 10^{r+8}}{5} = \frac{30}{5}$$

$$\log(10^{r+8}) = \log(6)$$

$$r+8 = \log_6$$

$$\frac{-8}{-8} \quad \frac{-8}{-8}$$

$$r = \log_6 -8 = -7.2218$$

$$12) \frac{-8 \cdot 12^{5p}}{-8} = \frac{-65}{-8}$$

$$\ln(12^{5p}) = \frac{\ln(-65)}{\ln(-8)}$$

$$\frac{5p \ln 12}{5 \ln 12} = \frac{\ln(-65/8)}{5 \ln 12}$$

$$p = .1686$$

$$13) \frac{4e^{2b-8}}{4} = \frac{94.7}{4}$$

$$\ln(e^{2b-8}) = \frac{\ln(94.7)}{4}$$

$$2b - 8 = \ln\left(\frac{94.7}{4}\right) + 8$$

$$\frac{+8}{+8}$$

$$\frac{2b}{2} = \frac{\ln(94.7) + 8}{2} \quad 5.5822$$

$$15) 6 \cdot 9^{2x-1} + 10 = 18$$

$$\frac{6 \cdot 9^{2x-1}}{6} = \frac{8}{6}$$

$$\frac{(2x-1)\ln 9}{\ln 9} = \frac{\ln\left(\frac{4}{3}\right)}{\ln 9}$$

$$X = .5655$$

$$14) \frac{15^{9a-7} + 8}{-8} = 38$$

$$\ln(15^{9a-7}) = \frac{30}{-8}$$

$$\frac{(9a-7)\ln 15}{\ln 15} = \frac{\ln 30}{\ln 15}$$

$$9a - 7 =$$

$$a = .9173$$

$$16) \frac{8 \cdot 19^{-8x-4.6} + 1}{-1} = 81$$

$$\frac{8 \cdot 19^{-8x-4.6}}{8} = \frac{80}{8}$$

$$\ln(19^{-8x-4.6}) = \frac{10}{\ln 19}$$

$$\frac{\ln(19^{-8x-4.6})}{\ln 19} = \frac{10}{\ln 19}$$

$$-8x - 4.6 = .78201148$$

$$X = -.6728$$

Solve each equation.

$$17) \log_3(2x+9) = \log_3 1$$

$$2x + 9 = 1$$

$$2x = -8$$

$$x = -4$$

$$18) \log_3 -2x = \log_3 (-5x - 6)$$

$$-2x = -5x - 6$$

$$3x = -6$$

$$x = -2$$

$$19) \log_{20}(2p - 4) = \log_{20} 30$$

$$2p - 4 = 30$$

$$2p = 34$$

$$p = 17$$

$$20) \log_{16}(-3x + 2) = \log_{16}(7 - x)$$

$$-3x + 2 = 7 - x$$

$$\frac{-5}{2} = \frac{2x}{2}$$

$$-\frac{5}{2} = x$$

$$21) \log_{12}(-3b + 2) = \log_{12}(b^2 + 4)$$

$$-3b + 2 = b^2 + 4$$

$$b^2 + 3b + 2 = 0$$

$$(b+2)(b+1) = 0$$

$$b = -2 \text{ or } -1$$

$$22) \log_{19}(18 - b) = \log_{19}(b^2 + 2b)$$

$$18 - b = b^2 + 2b$$

$$b^2 + 3b - 18 = 0$$

$$(b+6)(b-3) = 0$$

$$b = -6 \text{ or } 3$$

$$23) \frac{-9 + \log_5 7v}{+9} = -7$$

$$\log_5 7v = 2$$

$$7v = 5^2$$

$$7v = 25$$

$$v = 25/7$$

$$24) -8 \log_9 n = -16$$

$$\frac{-8 \log_9 n}{-8} = \frac{-16}{-8}$$

$$9 \log_9 n = 9^2$$

$$n = 81$$

$$n = 81$$

$$25) \log_8 10 - \log_8(x+5) = 2$$

$$\log_8 \frac{10}{x+5} = 2$$

$$\frac{10}{x+5} = 64$$

$$10 = 64(x+5)$$

$$10 = 64x + 320$$

$$-310 = 64x$$

$$x = \frac{-155}{32}$$

$$26) \log_7 x - \log_7(x-4) = \log_7 10$$

$$\log_7 \frac{x}{x-4} = \log_7 10$$

$$\frac{x}{x-4} = 10$$

$$x = 10x - 40$$

$$\frac{40}{9} = \frac{9x}{9} \quad x = \frac{40}{9}$$

$$27) \log_6 9 + \log_6(x^2 - 9) = 4$$

$$\log_6 9(x^2 - 9) = 4$$

$$\frac{9(x^2 - 9)}{9} = 1296$$

$$\frac{x^2 - 9}{9} = 144$$

$$\frac{+9}{+9} \quad x^2 = 153$$

$$x = \pm 3\sqrt{17}$$

$$28) \log_2 4x - \log_2 5 = 4$$

$$\log_2 \frac{4x}{5} = 4$$

$$5 \left(\frac{4x}{5} = 16 \right) 5$$

$$4x = 80$$

$$x = 20$$

Rewrite the following using the Change of Base formula.

29) $\log_5 68$

$$\frac{\log 68}{\log 5}$$

$$2.622$$

30) $\log_2 4.7$

$$\frac{\log 4.7}{\log 2}$$

$$2.233$$

Condense each expression to a single logarithm.

31) $\log_3 z + \frac{\log_3 x}{2} + \frac{\log_3 y}{2}$

$$\log_3 (z \sqrt{xy})$$

32) $5\log_2 a + 2\log_2 b$

$$\log_2(a^5 b^2)$$

33) $3\log_2 7 + \frac{\log_2 12}{2} + \frac{\log_2 11}{2}$

$$\log_2 7^3 \sqrt{12 \cdot 11}$$

$$\log_2 7^3 \sqrt{132}$$

34) $6\log_4 u + 18\log_4 v + 6\log_4 w$

$$\log_4(u^6 v^{18} w^6)$$

$$\log_4(uv^3w)^6$$

Expand each logarithm.

35) $\log_4(w^5 \sqrt[3]{u})$

$$5\log_4 w + \frac{1}{3}\log_4 u$$

36) $\log_3(u \cdot v \cdot w^5)$

$$\log_3 u + \log_3 v + 5\log_3 w$$

37) $\log_9(11 \cdot 3\sqrt{10 \cdot 3})$

$$\log_9 11 + \log_9 3 + \frac{1}{2}\log_9 3 + \frac{1}{2}\log_9 10$$

$$\log_9 11 + \frac{3}{2}\log_9 3 + \frac{1}{2}\log_9 10$$

38) $\ln(7 \cdot 2^4 \cdot 11)^4$

$$4(\ln 7 + 4\ln 2 + \ln 11)$$

$$4\ln 7 + 16\ln 2 + 4\ln 11$$

Evaluate each expression without a calculator.

39) $\log_5 125$

3

40) $\log_3 81$

4

41) $\log_2 32$

5

42) $\log_2 -64$

Undefined

Find the inverse of each function.

43) $y = \log_6 x - 5$

$$\begin{aligned}x &= \log_6 y + 5 \\x + 5 &= \log_6 y \\6^{x+5} &= 6^{\log_6 y} \\6^{x+5} &= f^{-1}(x)\end{aligned}$$

45) $y = \log_3 (3x)$

$$\begin{aligned}x &= \log_3 (3y) \\3^x &= 3^{\log_3 (3y)} \\3^x &= 3^{\log_3 3 + \log_3 y} \\3^x &= 3^{\log_3 3} \cdot 3^{\log_3 y} \\3^x &= 3 \cdot 3^{\log_3 y} \\3^x &= 3f^{-1}(x)\end{aligned}$$

47) $y = \log_4 5^x$

$$\begin{aligned}x &= \log_4 5^y \\4^x &= 4^{\log_4 5^y} \\4^x &= 4^{\log_4 5 + \log_4 y} \\4^x &= 4^{\log_4 5} \cdot 4^{\log_4 y} \\4^x &= 5 \cdot 4^{\log_4 y} \\4^x &= 5f^{-1}(x)\end{aligned}$$

44) $y = -10 \log_4 x$

$$\begin{aligned}x &= -\frac{10 \log_4 y}{-10} \\x &= \frac{10 \log_4 y}{10} \\x &= \log_4 y \\4^x &= 4^{\log_4 y} \\4^x &= f^{-1}(x)\end{aligned}$$

46) $y = \log_4 x - 2$

$$\begin{aligned}x &= \log_4 y + 2 \\x + 2 &= \log_4 y \\4^x &= 4^{\log_4 y} \\4^x &= 4^{\log_4 y - 2} \\4^x &= 4^{\log_4 y} \cdot 4^{-2} \\4^x &= 4^{\log_4 y} \cdot \frac{1}{16} \\4^x &= \frac{1}{16} 4^{\log_4 y} \\4^x &= \frac{1}{16} f^{-1}(x)\end{aligned}$$

48) $y = 5^x - 9$

$$\begin{aligned}x &= 5^y - 9 \\x + 9 &= 5^y \\log_5(x+9) &= y\end{aligned}$$

$$49) y = -\frac{6^x}{4}$$

$$x = -\frac{6^y}{4}$$

$$-4x = 6^y$$

$$\log_6(-4x) = y$$

$$50) y = -\frac{5^x}{3}$$

$$x = -\frac{5^y}{3}$$

$$-3x = 5^y$$

$$\log_5(-3x) = y$$

Rewrite each equation in exponential form.

$$51) \log_5 125 = 3$$

$$5^3 = 125$$

$$52) \log_{12} 144 = 2$$

$$12^2 = 144$$

$$53) \log 1000 = 3$$

$$10^3 = 1000$$

$$54) \log_{16} 2 = \frac{1}{4}$$

$$16^{\frac{1}{4}} = 2$$

Rewrite each equation in logarithmic form.

$$55) 3^4 = 81$$

$$\log_3 81 = 4$$

$$56) 8^2 = 64$$

$$\log_8 64 = 2$$

$$57) 13^2 = 169$$

$$\log_{13} 169 = 2$$

$$58) 20^2 = 400$$

$$\log_{20} 400 = 2$$

Identify the domain and range of each.

59) $y = \sqrt{x-3} + 5$

D: $[3, \infty)$

R: $[5, \infty)$

60) $y = -1 + \sqrt{x+5}$

D: $[-5, \infty)$

R: $[-1, \infty)$

Solve each equation. Remember to check for extraneous solutions.

$$\begin{aligned} 61) \sqrt{-70+17n} &= n & -70+17n &= 0 \\ -70+17n &= n^2 & 17n &= 70 \\ n^2 - 17n + 70 &= 0 & n &\geq \frac{70}{17} \\ (n-10)(n-7) &= 0 & n &\geq \frac{70}{17} \\ 10 \text{ or } 7 & & & \end{aligned}$$

$$\begin{aligned} 63) k &= \sqrt{6k+15} - 4 & 6k+15 &\geq 0 \\ (k+4)^2 &= 6k+15 & \frac{6k}{6} &\geq \frac{-15}{6} \\ k^2 + 8k + 16 &= 6k+15 & k &\geq -\frac{5}{2} \\ k^2 + 2k + 1 &= 0 & (k+1)(k+1) &\quad k = -1 \end{aligned}$$

$$65) \frac{1}{x^2} + \frac{5}{2x} = \frac{1}{2x^2} \quad \text{LCD } 2x^2 \quad x \neq 0$$

$$2 + 5x = 1$$

$$\frac{5x}{5} = \frac{-1}{5}$$

$$x = -\frac{1}{5}$$

$$\begin{aligned} 62) -2 &= -8 + \sqrt{m+6} & m &\geq 6 \\ +8 &+8 & (\sqrt{m+6})^2 &= 6^2 \\ m+6 &= 36 & m &= 30 \\ m &= 30 & & \end{aligned}$$

$$\begin{aligned} 64) \frac{-45}{-9} &= \frac{-9\sqrt{x-3}}{-9} & x &\geq 3 \\ (5 = \sqrt{x-3})^2 & & & \\ 25 &= x-3 & & \\ x &= 28 & & \end{aligned}$$

$$\begin{aligned} 66) 3 + \frac{4}{b+3} &= \frac{5}{b+3} & b &\neq -3 \\ 3(b+3) + 4 &= 5 & & \\ 3b + 9 + 4 &= 5 & & \\ 3b &= -8 & & \\ \frac{3b}{3} &= \frac{-8}{3} & & \\ b &= -\frac{8}{3} & & \end{aligned}$$

Solve each equation.

67) $|p-2| = 11$

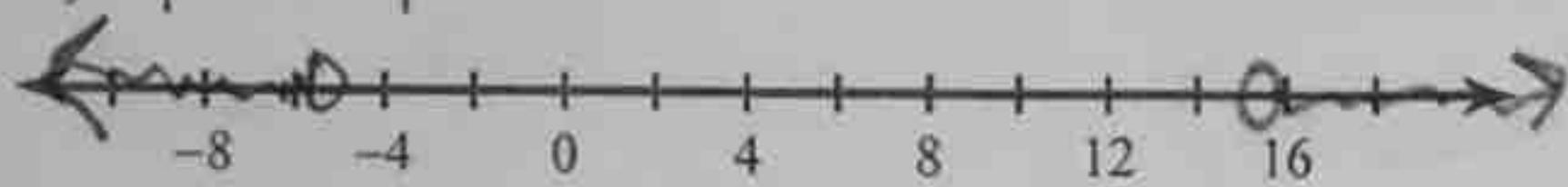
$$\begin{aligned} p-2 &= 11 & -(p-2) &= 11 \\ p &= 13 & -p+2 &= 11 \\ & & -p &= 9 \\ & & p &= -9 \end{aligned}$$

68) $|a-3| = 5$

$$\begin{aligned} a-3 &= 5 & -(a-3) &= 5 \\ a &= 8 & -a+3 &= 5 \\ & & -a &= 2 \\ & & a &= -2 \end{aligned}$$

Solve each inequality and graph its solution.

69) $|n - 5| > 10$



$$n - 5 > 10$$

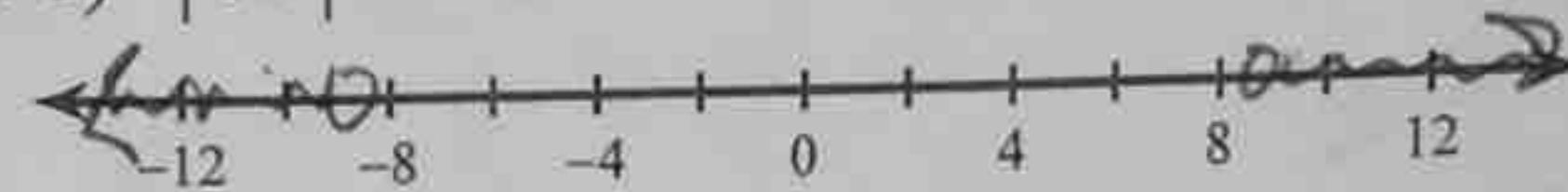
$$n > 15$$

$$-n + 5 > 10$$

$$-n > 5$$

$$n < -5$$

70) $|8n| > 72$



$$\frac{8n}{8} > \frac{72}{8}$$

$$n > 9$$

$$-8n > 72$$

$$n < -9$$

Find the inverse of each function.

71) $h(x) = \frac{6+3x}{2}$

$$x = \frac{6+3y}{2}$$

$$2x = 6+3y$$

$$\frac{2x-6}{3} = f^{-1}(x)$$

72) $g(x) = \frac{1}{x+1} - 2$

$$x = \frac{1}{y+1} - 2$$

$$(y+1)(x+2) = \frac{1}{y+1} (y+1)$$

$$y+1 = \frac{1}{x+2}$$

$$f^{-1}(x) = \frac{1}{x+2} - 1$$

Factor each and find all roots. One root has been given.

73) $x^4 + 2x^3 + 16x^2 - 2x - 17 = 0$; $-1 + 4i$

$$\begin{array}{r} -1+4i \\ \hline 1 & 2 & 16 & -2 & -17 \\ & \downarrow & -1+4i & -17 & 1-4i & 17 \\ \hline -1-4i & \downarrow & -1+4i & -1 & -1-4i & 0 \\ & & -1-4i & 0 & -1+4i & \\ \hline 1 & 0 & -1 & 0 & & \end{array}$$

$$x^2 - 1 = (x+1)(x-1)$$

roots $-1+4i, -1-4i, +1, -1$

$$\begin{array}{c|cc|c} x & x+1 & -4i \\ \hline x & x^2 & x & -4ix \\ -1 & x & 1 & -4i \\ 4i & 4ix & 4i & +16 \end{array}$$

$$x^2 + 2x + 17$$

$$(x-1)(x+1)(x^2 + 2x + 17)$$

74) $x^5 - 4x^4 - 15x^3 + 44x^2 - 54x + 168 = 0$; $4 + \sqrt{2}$

$$\begin{array}{r} 4+\sqrt{2} \\ \hline 1 & -4 & -15 & 44 & -54 & 168 \\ & \downarrow & 4+\sqrt{2} & 2+4\sqrt{2} & -44+3\sqrt{2} & 6+12\sqrt{2} & -168 \\ & & & & -48+12\sqrt{2} & 0 & \\ & & & & 12-3\sqrt{2} & & \end{array}$$

$$\begin{array}{r} 4-\sqrt{2} \\ \hline 1 & \sqrt{2} & -13+4\sqrt{2} & 3\sqrt{2} & -48+12\sqrt{2} & 0 \\ & \downarrow & 4-\sqrt{2} & 16 & -4\sqrt{2} & \\ & & & 12-3\sqrt{2} & 48 & -12\sqrt{2} \\ & & & & 12 & \\ & & & & 0 & \end{array}$$

$$\begin{aligned} & (x^3 + 4x^2) + (3x + 14) \\ & x^2(x+4) + 3(x+4) \end{aligned}$$

$$(x^2 + 3)(x+4)$$

$$x = 4 \pm \sqrt{2}, \pm i\sqrt{3}, -4$$

Fix ↗

SM3 Unit 6 Test Review Part 2

1. Circle all that are true. If $b > 1$, then $f(x) = b^x$ is:

Increasing

decreasing

always positive

always negative

2. Which point is common to every exponential function of the form $f(x) = b^x$? Explain for credit.

$$(0, 1)$$

anything to the
zero power is

3. Which point is common to every logarithmic function of the form $f(x) = \log_b x$? Explain for credit.

$$b^y = x$$

$$(1, 0)$$

inverse function

4. You have saved up some money. You decide to invest your money in a fund that compounds continuously at 5%. How many years does it take for you to triple the present amount?

$$\ln(3 = e^{.05t})$$

$$\frac{\ln 3}{.05} = \frac{.05t}{.05} \quad t = 21.97 \text{ yrs.}$$

5. Solve the inequality and write your answer in interval notation.

$$(2x - 1 \leq \sqrt{4x + 1})^2$$

$$4x + 1 \geq 0 \\ x \geq -\frac{1}{4}$$

$$4x^2 - 4x + 1 = 4x + 1$$

$$4x^2 - 8x = 0$$

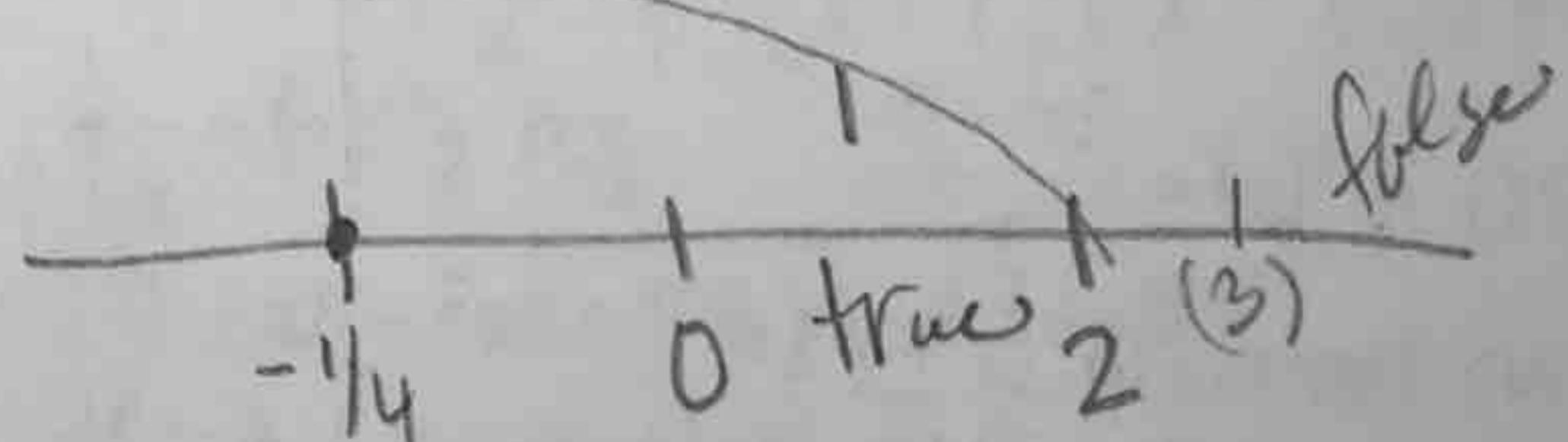
$$4x(x - 2) = 0$$

$$x = 0 \text{ or } x = 2$$

$$[-\frac{1}{4}, 2]$$

$$2 - 1 \leq \sqrt{4 + 1} \\ 2 - 1 \leq \sqrt{5}$$

$$3 \leq 5 \quad 6 - 1 \leq \sqrt{13}$$



6. For the following function state the following. $\frac{3x^3+4x^2-4x}{x^2-4}$

a. the y-intercept $(0, 0)$

b. the x-intercept $(\frac{2}{3}, 0)$

c. any vertical asymptotes $x = 2$

d. any horizontal or oblique asymptotes

e. any holes $\textcircled{a} -2$

$$\begin{array}{c} \cancel{-12} \\ \cancel{6} \end{array} \begin{array}{c} \cancel{0} \\ \cancel{-2} \end{array} \begin{array}{c} (x+\frac{2}{3})(x-\frac{2}{3}) \\ (x+2)(3x-2) \end{array}$$

oblique $3x+4$

$$\frac{x(x+2)(3x-2)}{(x-2)(x+2)}$$

$$\begin{array}{r} 3x+4 \\ x^2-4 \end{array} \begin{array}{r} 3x^3+4x^2-4x \\ 3x^3 - -12x \\ \hline 4x^2-8x \end{array}$$

$$\frac{x(3x^2+4x-4)}{(x-2)(x+2)}$$

7. Pickles and Peeps are starting a company selling poppers. They have derived a price equation, $p(x) = -x^2 + 50x$. The cost to produce x poppers is given by $C(x) = 3x + 25$. Write a function that represents their profit.

$$P(x) = -x^3 + 5x^2 - (3x + 25)$$

$$P(x) = -x^3 + 5x^2 - 3x - 25$$

8. The Smith's bought a boat for \$9,500 three years ago. If the boat depreciates at a rate of 4.5% per year, how much is the boat worth now?

$$A = 9500(1 - .045)^3$$

$$A = \$8,274.35$$

FYI: You will have a linear programming story problem on this test. This is similar to the linear programming problem you had on Unit Test 5.